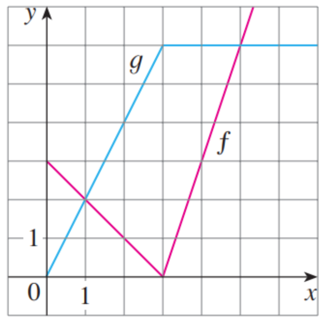
Exercise 70

If f and g are the functions whose graphs are shown, let $P(x) = f(x)g(x)$, $Q(x) = f(x)/g(x)$, and $C(x) = f(g(x))$. Find (a) $P'(2)$, (b) $Q'(2)$, and (c) $C'(2)$.



Solution

Part (a)

Find a formula for $P'(x)$.

$$\begin{aligned} P'(x) &= \frac{d}{dx}[f(x)g(x)] \\ &= \left[\frac{d}{dx}f(x)\right]g(x) + f(x)\left[\frac{d}{dx}g(x)\right] \\ &= f'(x)g(x) + f(x)g'(x) \end{aligned}$$

Plug in $x = 2$ to find $P'(2)$.

$$P'(2) = f'(2)g(2) + f(2)g'(2) = (-1)(4) + (1)(2) = -2$$

Part (b)

Find a formula for $Q'(x)$.

$$\begin{aligned} Q'(x) &= \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \\ &= \frac{\left[\frac{d}{dx}f(x)\right]g(x) - \left[\frac{d}{dx}g(x)\right]f(x)}{[g(x)]^2} \\ &= \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \end{aligned}$$

Plug in $x = 2$ to find $Q'(2)$.

$$Q'(2) = \frac{f'(2)g(2) - g'(2)f(2)}{[g(2)]^2} = \frac{(-1)(4) - (2)(1)}{(4)^2} = -\frac{3}{8}$$

Part (c)

Find a formula for $C'(x)$.

$$\begin{aligned} C'(x) &= \frac{d}{dx}\left[f(g(x))\right] \\ &= f'(g(x)) \cdot \left[\frac{d}{dx} g(x)\right] \\ &= f'(g(x)) \cdot g'(x) \end{aligned}$$

Plug in $x = 2$ to find $C'(2)$.

$$C'(2) = f'(g(2)) \cdot g'(2) = f'(4) \cdot 2 = 3 \cdot 2 = 6$$